Certainty Factor model

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The Certainty-Factor Model

- The certainty-factor(CF) model is a method for managing uncertainty in rule-based systems.
- avoid the unreasonable assumptions in Bayes-Model.

The Mechanics of the Model

The Holmers' brief

Mr. Holmes receives a telephone call from his neighbor Dr. Watson stating that he hears a burglar alarm sound from the direction of Mr. Holmes' house. Preparing to rush home, Mr. Holmes recalls that Dr. Watson is known to be a tasteless practical joker, and he decides to first call his other neighbor, Mrs.Gibbons, who, despite occasional drinking problems, is far more reliable.

We can find the rules:

- R_1 :if WATSON's CALL then ALARM, $CF_1 = 0.5$
- R_2 :if GIBBON's CALL then ALARM, $CF_1 = 0.9$
- R_3 :if ALARM's CALL then BURGLARY, $CF_1 = 0.99$

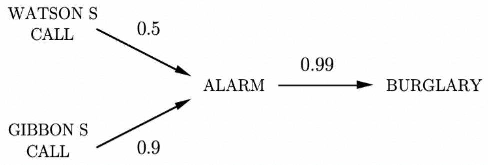


Figure 1: An inference network for Mr. Holmes' situation.

A CF represents a person's *change* in belief.

- The brief increases: a CF between 0 and 1
- The brief decreases: a CF between -1 and 0
- Combination functions: to CF that lie between the evidence and the hypothesis in question.

Step

1. combine CF_1 and CF_2
2. the CFs for R_1 and R_2 to give the CF for the rule R_4

- $R4$: If WATSON'S CALL and GIBBON'S CALL then ALARM, CF_4

Combine CF_1 and CF_2 using the function:

$$CF_4 = \begin{cases} CF_1 + CF_2 - CF_1 CF_2 & CF_1, CF_2 \geq 0 \\ CF_1 + CF_2 + CF_1 CF_2 & CF_1, CF_2 < 0 \\ \frac{CF_1 + CF_1}{1 - \min\{|CF_1|, |CF_2|\}} & otherwise \end{cases} \quad (1)$$

In the Mr.Holmes' case, we have

$$CF_4 = 0.5 + 0.9 - (0.5) \times (0.9) = 0.95$$

the equation (1) is called the parallel-combination function.

Generating another rules R_5: if WATSON'S CALL and GIBBON'S CALL then BURGLARY.

The combination function is

$$CF_5 = \begin{cases} CF_3 CF_4 & CF_3 > 0 \\ 0 & CF_3 \leq 0 \end{cases} \tag{2}$$

In Mr.Holmes' case, we have

$$CF_5 = (0.99) \times (0.95) = 0.94$$

This equation (2) called the serial-combination function.

The conjecture and Disconjecture

- R_6: if CHEST PAIN and SHORTNESS OF BREATH then HEART ATTACK, CF6 = 0.9 Further, suppose that we have indirect evidence for chest pain and shortness of breath:
- R_7: if PATIENT GRIMACES then CHEST PAIN, $CF_7 = 0.7$
- R_8: if PATIENT CLUTCHES THROAT then SHORTNESS OF BREATH, CF8 = 0.9

We can combine CF_6, CF_7, and CF_8 to yield the CF for the rule R9: if PATIENT GRIMACES and PATIENT CLUTCHES THROAT then HEART ATTACK, CF9

The combination function is

$$CF_9 = CF_6 \min(CF_7, CF_8) = (0.9) \min(0.7, 0.9) = 0.63$$

In general, the CF model prescribes that we use the minimum of CFs for evidence in a conjunction, and the maximum of CFs for evidence in a disjunction.

How about the Bayes Inference

Using the example of Mr.Holmes

WATSON'S CALL and GIBBON'S CALL are not conditionally independent given BURGLARY.

Theoretical Problems with CF Model

- Rules the represent logical relationship satisfy the principle of modularity.
- **principle of detachment:** no matter how we established that e is true
- **principle of locality**:no matter what else we know to be true.

Multiple Causes of the Same Effect

- Uncertain reasoning often violates the principles of detachment and locality.
- The inference network does not capture an important interaction among the propositions.

An Earthquake Example

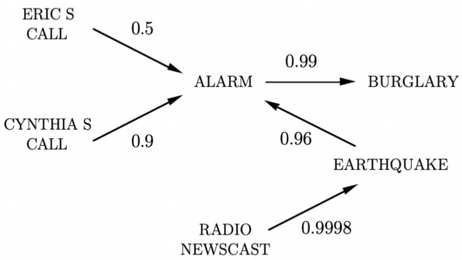


Figure 2: Another inference network for Mr. Holmes' situation.

- R_{11}: if RADIO NEWSCAST then EARTHQUAKE, $CF_{11} = 0.9998$
- R_{12}: if EARTHQUAKE then ALARM, $CF_{12} = 0.95$

He will decreases his brief that there has been a burglary, because the occurrence of an earthquake would account for the alarm sound.

(Pearl 2009) divides uncertain inference into diagnostic and predictive.

- diagnostic: given an effect changing the belief in a cause.
- predictive: given a cause changing the belief in an effect.
- the diagnostic inference can pass through to anther diagnostic inference.
- the predictive inference should not pass through to anther diagnostic inference.

Repair the inference by Adding the Rule

- R_{13}: if EARTHQUAKE then BURGLARY, $CF_13 = -0.7$

Ideally, we would like a representation that encodes only direct relationships among propositions.

Another problem

A Probabilistic Interpretation for Certainty Factors

We can using the likelihood ratio λ

$$\lambda = \frac{o(e|h,\xi)}{p(e|NOTh,\xi)}$$

The $p(e|h,\xi)$ denotes the probability that e is truely given that h is true.

And ξ denotes the background knowledge of the person to whom the belief belongs.

Using Bayes' Theorem

$$\lambda = \frac{O(h|e,\xi)}{O(h|\xi)} = \frac{\frac{p(h|e,\xi)}{1-p(h|e,\xi)}}{\frac{p(h|\xi)}{1-p(h|\xi)}}$$

CF model

$$CF = \begin{cases} \frac{\lambda - 1}{\lambda}, \lambda \geq 1 \\ \lambda - 1, \lambda < 1 \end{cases}$$

- The parallel-combination function follow exactly from Bayes' theorem. The serial-combination are close approximations to it.
- The parallel-combination function to combine CFs for the rules "if e_1 then h" and "if e_2 then h" based on CI(conditionally independent),given h and NOT h.
- Similarly, serial-combination function to CFs for the rules "is a then b" and "if b then c". CI, given b and NOT b.

The assumption of independence are not satisfied by real-world domains. In the parallel-combination function are stronger than Bayes inference.

$$H_0 = \theta_1, H_1 = \theta_2, \theta_1, \theta_2 \in \Theta$$

$$\theta_1 \cup \theta_2 = \Theta, \theta_1 \cap \theta_2 = \phi$$

$$H_0' = \theta_1, H_1' = \theta_i$$

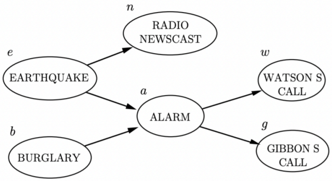
- The CF model could sole the one-to-one problem.
- Hard to solve one-to-n problem

A practical Problem with CF model

- a inference network must trace a trail of rules from observable evidence to hypothesis.
  - (Tversky and Kahneman 1982) found that people are usually most comfortable when they assess predictive rules.
  - the expert physicians prefer to asses the likelihood of a symptom, given a disease.
  - CF model effects are usually observable pieces of evidence.

Belief Network

$$p(e, b, a, n, w, g|\xi) = p(e|\xi)\cdot p(b|\xi)\cdot p(a|e, b, \xi)\cdot p(n|e, \xi)\cdot p(w|a, \xi)\cdot p(g|a, \xi)$$



$p(b+ \,|\, \xi) = 0.0001$

$p(e+ \,|\, \xi) = 0.0003$

$p(w+ \,|\, a-, \xi) = 0.4$
$p(w+ \,|\, a+, \xi) = 0.8$

$p(g+ \,|\, a-, \xi) = 0.04$
$p(g+ \,|\, a+, \xi) = 0.4$

$p(n+ \,|\, e-, \xi) = 0.0002$
$p(n+ \,|\, e+, \xi) = 0.9$

$p(a+ \,|\, b-, e-, \xi) = 0.01$
$p(a+ \,|\, b+, e-, \xi) = 0.95$
$p(a+ \,|\, b-, e+, \xi) = 0.2$
$p(a+ \,|\, b+, e+, \xi) = 0.96$

Figure 4: A belief network for Mr. Holmes' situation.

Belief Network and CF Model

- A knowledge provider can choose the order in which he prefers to assess probability distributions.
- Using a Belief network, the knowledge provider can control the assertions of conditional independence that are encoded in the representation. CF model forces a person to adopt assertions of conditional independence that may be incorrect.
- Using a belief network, a knowledge provider does not have to assess indirect independence.

Conclusion

- CF model as a useful approach to uncertainty management.
- CF model was created to avoid the unreasonable in Bayes model.
- But CF model does not perform well in real-world.
- The parallel-combination has a stronger assumption than conditional Independence in Bayes.
- Using different models to solve the different problems based on the characteristic of problems.

Thanks For your attention.

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